STRAIN ENERGY PROPORTIONAL DAMPING FOR MODAL FREQUENCY RESPONSE MODELS

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ABSTRACT

When solving for a frequency response, structural damping is modeled as the imaginary part of the stiffness matrix. This operation can be done on a nodal basis for a direct frequency response or on a modal basis for a modal frequency response. For the latter, since the stiffness matrix is projected on the modal basis, damping must then also be projected on the imaginary part of the modal basis.

Generally, accounting for structural damping in modal frequency response models is done in one of three ways: spatially uniform damping where the imaginary part of the dynamic stiffness matrix is directly proportional to its real part. As this operation can be done at every frequency, damping for this model can be frequency dependent. For spatially non-uniform damping, the damping must be projected on a modal basis. If the damping is non-frequency dependent, the finite element solver can project localized structural damping using the modal basis and the nodal stiffness matrix. This method usually implies that the localized damping cannot be frequency dependent. For frequency-dependent damping, one must devise a method to distribute damping on a modal basis at each frequency. At this stage of the calculation, the nodal stiffness matrix is usually not available. Localized damping can however be projected on a modal basis based on a chosen quantity. Commercial software solutions such as VA One have used the nodal mass matrix, leading to a damping distribution proportional to the kinetic energy on a modal basis. This method, however, assumes the damping is directly proportional to the kinetic energy in the model. Over the full mode shape, this is a reasonable assumption as the potential and kinetic energy are equal at resonance. However, in specific cases where damping is hyperlocalized and the number of modes is low, the modal projection using kinetic energy distribution does not represent the damping distribution on the modal basis accurately.

This paper proposes a new method to distribute damping on a modal basis using modal strain energy. The method gives results identical to the stiffness matrix projection performed by the finite element solver, except that it now allows for frequency-dependent damping. A review of all three damping distribution techniques is presented. Then, an example payload model is studied showing a results comparison for each damping distribution technique and is compared to a direct frequency response solution.

1. INTRODUCTION

When predicting the structural response of a test article in a dynamic environment, damping is often a variable of adjustment with flexibility and multiple possible definitions. In this context, damping represents the dissipation of energy intrinsic to the vibrating structure.

While damping is a generic term representing the intrinsic energy dissipation of a vibrating structure, it materializes through several phenomena: mainly, rubbing between two components or energy dissipation intrinsic to the material being deformed. The topic has been extensively studied, leading to many different damping models available in the literature. For example, one can opt to use either structural or viscous damping, uniform throughout the structure or non-uniform, or even frequency-dependent or not. However, we must note that, often, the abilities of the available damping models surpass the available information. Commonly, in industrial models, generic damping schedules are employed. This often ignores specificities of the physical model, sometimes even simplifying the formulation to the point of defining a single damping value, constant for the whole model and the whole frequency range. While this can be an option if conservative predictions are desired, it is certainly not the most accurate representation of the actual structure.

For industrial models, structural damping is commonly employed as it can easily be characterized and is defined as the imaginary part of the stiffness matrix, allowing for both non-uniform and frequency-dependent damping. However, we must note once again that, even though the employed model offers a lot of freedom to define damping, the corresponding necessary information may not be available at the time of solving.

Specifically, in the case of the modal frequency response, the damping information, potentially spatially non-uniform and frequency dependent, must be projected on a modal basis. Although simple in appearance, this last part often requires a dedicated methodology as the nodal stiffness matrix is often not available when damping projections on the modal basis are performed. Multiple methods have been proposed and studied in the literature [1] [2] [3] [4] [5].

This paper gives a review of the available structural damping projection models currently available in the VAOne software and details a proposed implementation of the strain energy damping distribution model. Finally, an example structure submitted to a random diffuse acoustic field is studied where the effect of the different damping models are compared to one another.

2. AVAILABLE DAMPING PROJECTION MODELS

Typically, modal frequency response solutions have limited damping modeling options. As damping must be expressed on a modal basis, the two simplest ways of accounting for damping in a simulation model are:

- to assume the damping uniform throughout the model,
- to project from the stiffness matrix as the normal mode analysis as performed by the finite element solver.

However, one may also choose a modal quantity of reference and project damping values at every frequency the modal is solved. This strategy is developed below.

2.1. Overview of the modal frequency response

Fundamentally, the dynamic behavior of a structural system is described by a frequency-dependent dynamic stiffness matrix $\boldsymbol{D}(\omega)$ linking the displacement response vector $\boldsymbol{u}$ to the force vector $\boldsymbol{f}$:

$$\mathbf{D}(\omega)\mathbf{u} = \mathbf{f} \qquad (1)$$

The formulation can be adapted for random vibration with the following:

$$\mathbf{D}(\omega)^{\boldsymbol{H}}\boldsymbol{S_{uu}}\mathbf{D}(\omega) = \boldsymbol{S_{ff}} \qquad (2)$$

where $\boldsymbol{S_{uu}}$ and $\boldsymbol{S_{ff}}$ represent the cross-spectral response and the cross-spectral loading respectively. For notation simplicity, the following will use the deterministic notation.

The real part of the dynamic stiffness matrix is built with the static stiffness matrix $\boldsymbol{K}$ and mass matrix $\boldsymbol{M}$ for a given angular frequency ω:

$$\mathbf{D}(\omega) = \mathbf{K} - \omega^2\mathbf{M} \qquad (3)$$

The modal frequency response projects each quantity from the nodal basis to the modal basis through the vector of the mode shapes $\boldsymbol{P}$.

$$\begin{aligned} \mathbf{K_q} &= \mathbf{P^T K P} \\ \mathbf{M}_{\mathrm{q}} &= \mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P} \\ \mathbf{f}_{\mathrm{q}} &= \mathbf{P}^{\mathrm{T}}\mathbf{f} \end{aligned} \qquad (4)$$

The dynamic stiffness matrix becomes:

$$[\boldsymbol{K_q} - \boldsymbol{\omega^2 M_q}]\boldsymbol{q} = \boldsymbol{f_q} \qquad (5)$$

with the response at any point $u(x)$ is expressed as

$$u(x) = \sum P_i(x) q_i \qquad (6)$$

Using both the orthogonality of the mode shapes and modes normalized to a unit of the generalized mass, we can write:

$$[\omega_i^2 - \omega^2]\boldsymbol{q} = \boldsymbol{f}_i \qquad (7)$$

with ω_i "is the i−th natural frequency" of the structure.

Overall, this means that the modal frequency response, when performed outside of the finite element solver is performed with only the natural frequencies ω_i and the corresponding mode shapes vector $\boldsymbol{P}$. In this, the structural damping is still expressed at the imaginary part of the stiffness matrix. Let $\mathbf{D}_{loss}$ represent the imaginary part of the dynamic stiffness matrix, the introduction of the loss matrix becomes

$$[\omega_i^2 + i\mathbf{D}_{loss} - \omega^2]\boldsymbol{q} = \boldsymbol{f}_i \qquad (8)$$

$\mathbf{D}_{loss}$ will then vary for each damping projection model. From now on, this paper will focus on the different projection models used to describe $\mathbf{D}_{loss}$.

2.2. Spatially uniform damping

Although the simplest, a spatially-uniform damping model is also the most commonly used. In this formulation, the modal damping is set to be directly proportional to the modal stiffness matrix.

$$\mathbf{D}_{loss}(\omega) = \eta(\omega)\,\mathbf{K}_{modal}$$
$$\mathbf{D}_{loss}(\omega) = \eta(\omega)\,\omega_i^2 \qquad (9)$$

This formulation does allow for frequency dependence and is accommodating well the damping schedules mentioned in the introduction.

2.3. Imported modal damping

Similarly to the operations performed in equation (4), most finite element solvers also project the imaginary part of the nodal stiffness matrix to the modal stiffness matrix. Therefore, we have:

$$\mathbf{D}_{loss} = \mathbf{P}\,\mathrm{Im}(\mathbf{K})\,\mathbf{P}^{\mathrm{T}} \qquad (10)$$

The advantage here is the ability to define spatially non-uniform damping, however, in this case, $\mathbf{K}$ is not frequency dependent, and there $\mathbf{D}_{loss}$ is not frequency dependent.

2.4. Kinetic energy proportional damping

To accommodate both frequency-dependent and spatially non-uniform damping, the modal damping projection has to be performed at every frequency during the solving process with a quantity of reference. The quantity should vary for every mode and spatially. Initial implementations in VA One used the kinetic energy for this process, in which case $\mathbf{D}_{loss}$ becomes

$$\mathbf{D}_{loss}(\omega) = \left[i\eta_s(\omega)\omega_{n,s}^2\right]$$
$$\text{with } \eta_s(\omega) = \frac{\sum_{\text{FE Subs},m} \eta_m(\omega)\mathbf{P}_s^{\mathrm{T}}\mathbf{m}_m\mathbf{P}_s}{\sum_{\text{FE Subs},m} \mathbf{P}_s^{\mathrm{T}}\mathbf{m}_m\mathbf{P}_s} \qquad (11)$$

for mode s and subsystem m. The validity of this assumption relies on the fact that it is expected for the kinetic energy to be proportional to the strain energy. Validation studies have shown that this assumption is valid when subsystems have a large number of modes. However, it typically finds its limitation when damping is hyper-localized on a model and greater care must be taken when projecting damping on a modal basis.

Effectively, this method determines unique values of modal damping for each frequency at which we are solving. This allows for frequency-dependent damping.

2.5. Strain energy proportional damping

To project spatially non-uniform and frequency-dependent damping onto the modal basis with the same accuracy as in equation (10), a newly implemented method distributing the damping proportionally to the modal strain energy is proposed. The method can then evaluate frequency-dependent modal damping distributed similarly to the projection done by the finite element solver. With this, the diagonal terms of $\mathbf{D}_{loss}$ are equal to those obtained by the finite element solver for a given frequency. The proposed formulation is then written as

$$\mathbf{D}_{loss}(\omega) = \left[i\eta_s(\omega)\omega_{n,s}^2\right]$$
$$\text{with } \eta_s(\omega) = \frac{\sum_{\text{FE Subs},m} \eta_m(\omega)\mathbf{E}_{s,m}^{\varepsilon}}{\sum_{\text{FE Subs},m} \mathbf{VE}_{s,m}^{\varepsilon}} \qquad (12)$$

and $\mathbf{VE}_{s,m}^{\varepsilon}$ is the total strain energy of mode s and subsystem.

The challenge of the implementation of this method is to make available the strain energy proportion for each mode available into a separate solver outside of the finite element solver without exporting the whole stiffness matrix which can be cumbersome.

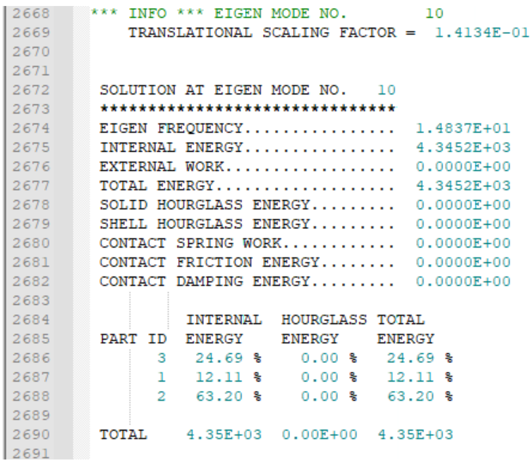
For this, some finite element solvers, such as ESI's Virtual Performance Solution, can output the proportion of modal strain energy for each PID and each mode. This proportion can be expressed as a percentage of the modal strain energy, $p_{\mathbf{E}_{s,m}^{\varepsilon}}$, defined as

$$p_{\mathbf{E}_{s,m}^{\varepsilon}} = \frac{\mathbf{E}_{s,m}^{\varepsilon}}{\sum_{\text{FE Subs},m} \mathbf{VE}_{s,m}^{\varepsilon}} \qquad (13)$$

Then equation (12) becomes

$$\mathbf{D}_{loss}(\omega) = \left[i\eta_s(\omega)\omega_{n,s}^2\right]$$
$$\text{with } \eta_s(\omega) = \sum_{PID,m} \eta_m(\omega)\,p_{\mathbf{E}_{s,m}^{\varepsilon}} \qquad (14)$$

As mentioned above, $p_{\mathbf{E}_{s,m}^{\varepsilon}}$ is directly available in the log file as shown in Figure 1 which makes the implementation and prototyping of this damping projection model implementation simple.



```
*** INFO *** EIGEN MODE NO.         10
    TRANSLATIONAL SCALING FACTOR =  1.4134E-01


 SOLUTION AT EIGEN MODE NO.   10
 ******************************
 EIGEN FREQUENCY................  1.4837E+01
 INTERNAL ENERGY................  4.3452E+03
 EXTERNAL WORK..................  0.0000E+00
 TOTAL ENERGY...................  4.3452E+03
 SOLID HOURGLASS ENERGY.........  0.0000E+00
 SHELL HOURGLASS ENERGY.........  0.0000E+00
 CONTACT SPRING WORK............  0.0000E+00
 CONTACT FRICTION ENERGY........  0.0000E+00
 CONTACT DAMPING ENERGY.........  0.0000E+00

          INTERNAL  HOURGLASS TOTAL
 PART ID  ENERGY    ENERGY    ENERGY
       3   24.69 %    0.00 %   24.69 %
       1   12.11 %    0.00 %   12.11 %
       2   63.20 %    0.00 %   63.20 %

 TOTAL    4.35E+03  0.00E+00  4.35E+03
```

Figure 1 - Strain energy per part ID in a finite element solver log file. The strain energy here is

referred at internal energy.

Similarly to the kinetic energy proportional damping, this method effectively calculates unique values of modal damping for each frequency we are solving. It is worth noting that, for the same values of damping at a given frequency, the modal damping calculated with this method is identical to the imported modal damping method discussed in 2.3.

3. EXAMPLE PAYLOAD AND DAMPING MODEL COMPARISON

3.1. Reference model and input data

So to compare the different damping projection models presented in section 2, a generic satellite structure submitted to a diffuse acoustic field modeled with the boundary element method. This type of model is an industry standard and has been presented and reviewed extensively [6] [7] [8]. While being a fully coupled model, the response of the structure has the form of a modal frequency response, and therefore, damping must be defined on a modal basis.

In this model, each part ID is assumed to have a well-characterized material damping as a function of the frequency.

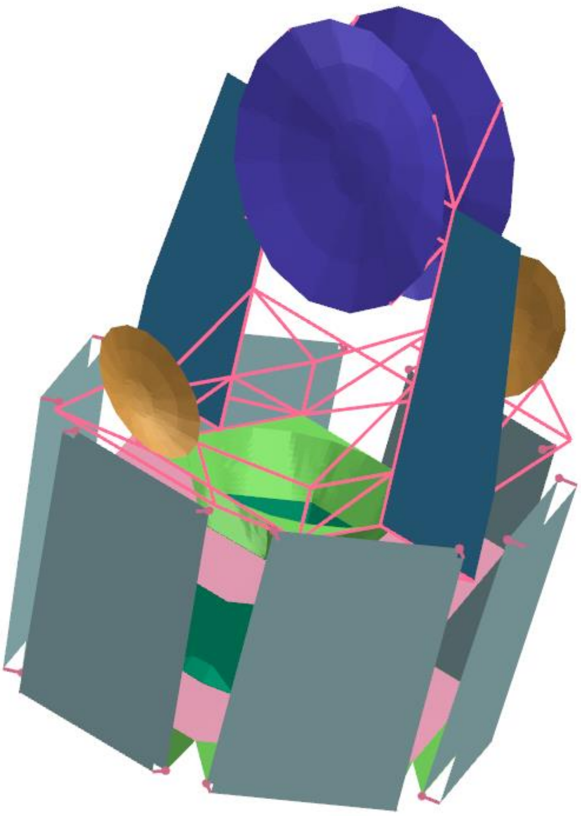


Figure 2 - Studied satellite structure

The damping spectrum of each material construction is described in Figure 3 and the corresponding location on the structure is shown in Figure 4.

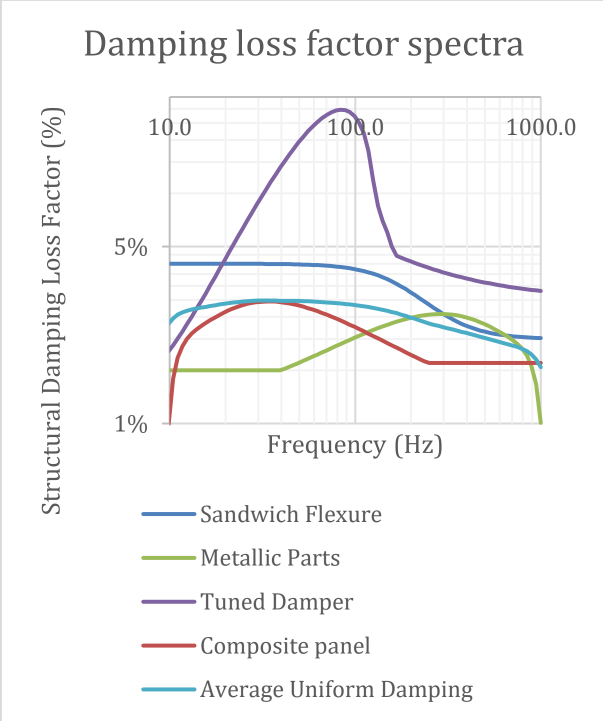


Figure 3 - Characterized damping loss factor spectra for each construction

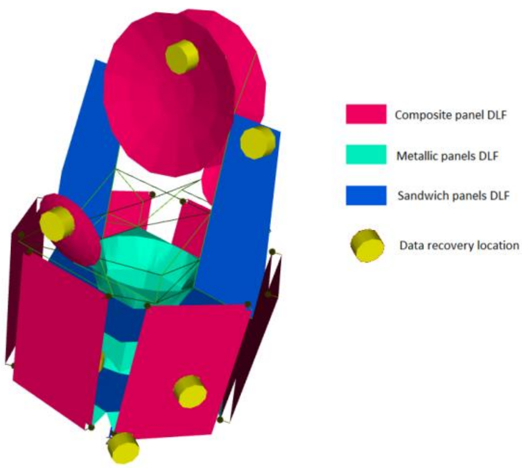


Figure 4 - Construction description of the representative structure

Figure 3 also shows the average damping spectrum used for the spatially uniform damping model described in 2.2. For the imported modal damping, the finite element solver is using a single constant value per part ID over the frequency range. Typically, an average value over a given frequency range is used. For this study, the values in Table 1 are used for this model.

Table 1 - Frequency average values for each part for the imported damping model

	Sandwich Flexure	Metallic Parts	Tuned Damper	Composite panel
Average damping value up to 200Hz	3.80%	1.25%	12.14%	1.89%

Figure 4 also shows 6 key data recovery locations (named Reflector – 1, Reflector – 2, Sandwich – 1, Sandwich – 2, Solar Array – 1, Solar Array – 2) where the structural response is recovered and used for comparison.

3.2. Single frequency point implementation verification

To validate the implementation of the projection algorithm we can set a special version of the imported modal damping such that the damping set on the finite element model match the damping values from the spectra defined in Figure 3 at a given frequency. We then expect that the corresponding imported modal damping matches exactly the strain-energy damping. For this verification, we are choosing the 127.43 Hz frequency for this exercise as we know the response of the solar array is very different from the strain-energy proportional damping model, and therefore different from the other methods. Table 3 shows a summary of the calculated modal damping at 127.43 Hz, with the first three modes and the last mode. The green section of the table are showing the minima et maxima of the corresponding columns as well as the RMS values of those columns. We can see that the calculated modal damping is similar for both methods (the RMS of the damping value difference between both methods is under 2%). Minor differences are attributed to the number of digits present in the log file (see Figure 8) for the strain energy distribution as well as an interpolation function used for the damping loss factors in the code.

Table 2 - Damping values at 127.43 Hz from Figure 3

Hz	Sandwich Flexure	Metallic Parts	Tuned Damper	Composite panel
Damping value at 127.43 Hz	3.51%	1.69%	11.82%	1.52%

Table 3 - Modal damping comparison at 127.43 Hz

		Modal Damping from Projection algorithm	Modal damping from Finite Element Solver	Difference
Mode #	Maximum	0.0876	0.0972	3.49%
	Minimum	0.0155	0.0151	-10.99%
	RMS	0.0200	0.0202	1.81%
1		0.0168	0.0167	0.84%
2		0.0168	0.0166	0.84%
3		0.0168	0.0167	0.83%
…		…	…	…
3244		0.0162	0.0161	0.89%

3.3. Results comparison

Figure 5 shows responses at location “Sandwich 1” for all 4 damping projection models. We first notice all 4 response curves are fairly similar in terms of trend. Differences between curves are localized to given frequencies where either a given mode is active or a frequency spectrum has a higher value of damping.

Out of all four methods, the Strain Energy Proportional damping gives results that differ the most from the other three methods. This is because the strain energy method allows for an accurate projection of the local damping values on a modal basis while allowing for frequency-dependent damping. This model illustrates this well as the damping here is both very localized and frequency dependent.

We also observe that as the frequency increases, responses from all four methods are closer to each other. This observation is particularly true when comparing the kinetic energy proportional damping and the strain energy proportional damping, as it is assumed that both methods trend to similar results as the frequency and modal density increase.

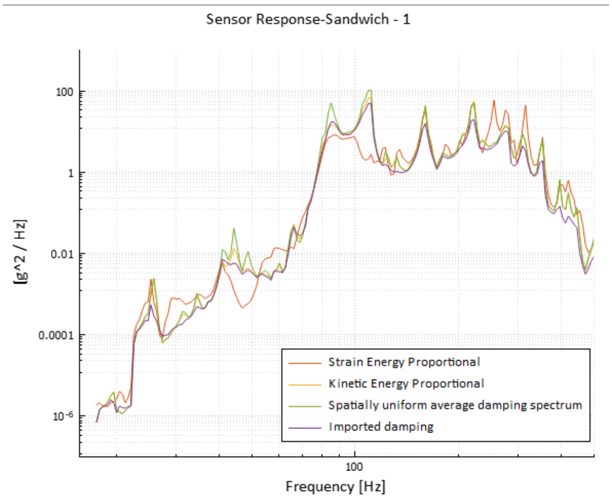


Figure 5 - Sandwich 1 location structural response comparison for the four damping distribution models

These observations can be reproduced on other sensor locations in Figure 6. While Figure 7 reproduces some of these observations, the results convergence between the strain energy and kinetic energy proportional damping project models is less obvious and may need to be investigated in the future.

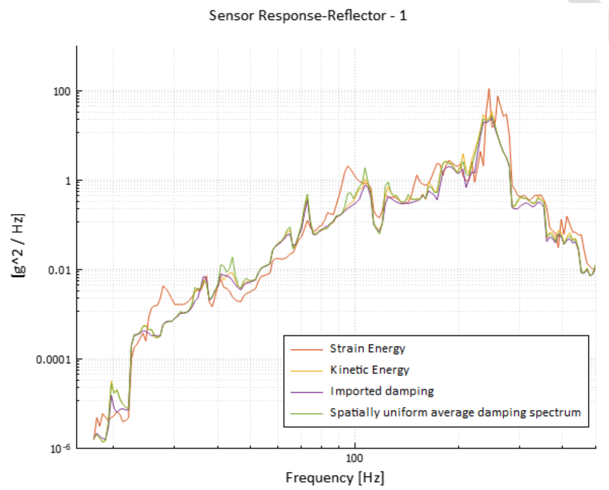


Figure 6 - Reflector 1 location structural response comparison for the four damping distribution models

Figure 7 - Solar Array 2 location structural response comparison for the four damping distribution models

Figure 5 shows the clearest differences between the proposed projection models. At about 47Hz, the response difference is very clear. When observing the structural mode shapes at this frequency, we can see that the solar array of the studied structure is particularly active. Additionally, the solar array is connected to the main bus through a truss where the tune damper damping loss factor is connected.

Figure 8 confirms this observation showing a concentration of the modal strain energy on the composite panels at this 47 Hz frequency.

Figure 7 shows a much higher response of the composite panels at higher frequencies. This is expected as this panel isn't stiffened and has very localized low damping at (1.10%).

```
SOLUTION AT EIGEN MODE NO.  208
*****************************
EIGEN FREQUENCY...............  4.8933E+01
INTERNAL ENERGY...............  4.7037E+04
EXTERNAL WORK.................  0.0000E+00
TOTAL ENERGY..................  4.7037E+04
SOLID HOURGLASS ENERGY........  0.0000E+00
SHELL HOURGLASS ENERGY........  0.0000E+00
CONTACT SPRING WORK...........  0.0000E+00
CONTACT FRICTION ENERGY.......  0.0000E+00
CONTACT DAMPING ENERGY........  0.0000E+00


E N E R G I E S   P E R   P A R T S
PART ID   INTERNAL   HOURGLASS INTERNAL
                               TOTAL
 100000      0.94 %     0.00 %     0.94 %
 100005      0.00 %     0.00 %     0.00 %
 100009     96.94 %     0.00 %    96.94 %
 100011      2.12 %     0.00 %     2.12 %

TOTAL      4.70E+04   0.00E+00   4.70E+04
```

Figure 8 - Strain energy distribution for mode 208. Here 96.9% of the strain energy is on the compositive panels (PID 100009).

4. CONCLUSIONS

Generally, all four damping models give similar results. Some differences are observed when large differences in damping values between the subsystems are used at a given frequency. One may choose to attribute differences between the different models to modeling uncertainties. However, given the nature of each model, we can determine which is the most accurate.

The summary review of the different damping projection models is presented in Table 4.

Table 4 Damping model review table

	Spatially uniform damping	Imported modal damping	Kinetic energy proportional damping	Strain energy proportional damping
Damping can be frequency dependent	Yes	No	Yes	Yes
Damping can vary spatially	No	Yes	Yes	Yes
Assumption on damping projection model	No	No	No	No

Though all four models give similar trends, the strain energy proportional damping is a formulation without compromise. This model is the most accurate as it does not make any compromise between accuracy and flexibility. Its current implementation uses strain energy distribution per part as output by the finite element solver.

As this quantity may not be standard on all finite element solvers, at the time of writing this paper, alternate implementation options of the strain energy proportional damping are being considered such that limited outputs from the finite element solver would be necessary for this formulation.

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